Active Structural Acoustic Control of
Clamped Flat Plates using a Weighted Sum of Spatial Gradients.

William R. Johnson, Daniel R. Hendricks, and Jonathan D. Blotter
Department of Mechanical Engineering,
Brigham Young University,
Provo,
Utah
84602

Scott D. Sommerfeldt, and Kent L. Gee
Department of Physics and Astronomy,
Brigham Young University,
Provo,
Utah
84602
Abstract

In this paper, the weighted sum of spatial gradients, or WSSG (formerly known as composite velocity) is developed for use in active structural acoustic control (ASAC) on clamped flat rectangular plates. In previous work, WSSG was developed on a simply-supported flat rectangular plate\cite{pseudo}, and showed promise as a control metric. The displacement on the clamped plate has been modeled using an approximate analytical solution assuming shape functions corresponding to clamped-clamped beams. From the analytical formulation, weights, which were found to be the reciprocal of the wave number squared, have been derived to produce a uniform WSSG field across the plate. In active control simulations this quantity has been found to provide improved control compared to volume velocity. Sensitivity analysis has also shown that comparable control, regardless of the sensor location, can be achieved using WSSG as an objective function. Experimental results have demonstrated that this control metric works effectively in practice, with results similar to the simulations. The results show that WSSG can be used as an effective control metric on clamped rectangular plates.
I. Introduction

Active structural acoustic control (ASAC) is an important subfield of active noise control (ANC). In ASAC, sound fields radiated by structures are controlled by using a control actuator applied to the vibrating structure as opposed to being placed in the sound field\(^2\). As research in ANC increased in the 1980's it was noted that for sound control of vibrating plates the number of actuators (speakers) in a sound field required to achieve global control was frequency dependent\(^3\). It was also noted alternatively that only one or two control forces applied to the structure\(^4\), as opposed to multiple actuators placed in the sound field, could provide global sound reduction. This led to the suggestion of using an active control algorithm to control actuators on the structure\(^5\) for improved control of global sound radiation and simplification of the control hardware.

Initial attempts at ASAC used error microphones placed in the radiated sound field. By moving the actuator to the surface of the plate significant global reduction, by as much as 40 dB in some cases, was achieved while avoiding the problem of an increasing number of actuators in the sound field. It was shown that one or two actuators were needed in order to provide control regardless of the frequency of excitation. This was due to the fact that only one to two structural modes seemed to contribute to radiated sound at a given frequency\(^6\).

Another important step for ASAC was the development of the radiation resistance matrix method which provided a simple way to calculate the radiated sound power of a distributed system. One of the key analytical tools in the development of ASAC was derived from this, namely the acoustical or radiation modes of a structure, which are the eigenvectors of the radiation resistance matrix. The radiation modes are a set of orthogonal modes which radiate sound power independently\(^2\). Each radiation mode is composed of several contributing structural modes and can be used to show which structural modes are significant in the radiation of sound power\(^7,8\).

For rectangular plates it was shown that for values of \(k l < 0.5\) where \(k\) is the wave number and \(l\) is the length of the longer side of the plate, the first radiation mode is the only radiation mode which radiates substantially. The shape of this first mode is comparable to volume velocity and suggested that using an error sensor to measure volume velocity on the plate would be an effective way to control radiated sound in the sound field\(^8\). This was significant in that it removed the error sensor as well as the control actuator from the sound field and placed them both on the plate, thus allowing for control of a sound field while keeping it clear of measurement or control devices.

With this idea in mind, sensing techniques for volume velocity were investigated. Two of the methods pursued were accelerometer arrays\(^9\) and distributed piezoelectric films\(^10\). An accelerometer array can give an accurate estimate of the volume velocity of the plate. However, it requires a large number of sensors to be implemented. For example, on a plate with dimensions of 0.278 m x 0.247 m it was predicted that 16 to 25 accelerometers would be needed to effectively measure the volume velocity\(^9\).
With larger plates this number increases. Making a measurement in this manner does not reduce the complexity of the hardware required for control, thus eliminating one of the potential benefits of ASAC.

A distributed piezoelectric film avoids the necessity of a large number of error sensors to sense volume velocity, requiring only a single piezoelectric patch. In this case the difficulty is not in the number of sensors required but in the fact that the sensor needs to be designed specifically for the geometry in question. For a plate of size 0.314 m x 0.414 m a PVDF film was used with dimensions of 0.306 m x 0.406 m\(^2\). For a different size plate an entirely different sensor would need to be designed and built. Thus while effective, this method is highly geometry dependent and difficult to implement.

Other methods were developed for sensing radiation mode shapes, such as optimally shaped piezoelectric films designed to target the specific structural vibration modes contributing significantly to the important acoustic radiation modes\(^1\). This allowed for the first radiation mode to be targeted specifically as well as several other higher but significantly radiating modes to be targeted. Structural energy methods have also been investigated\(^2\), but with limited success.

Optimally shaped piezoelectric films have the benefit of targeting the radiation modes of a structure more directly. Because only a few radiation modes contribute to sound radiation at low frequencies, and only a few structural modes contribute significantly to each radiation mode this method requires a smaller number of sensors than volume velocity. If the first radiation mode were the only mode of interest (also the target of volume velocity), three to four sensors would be required, depending on the frequency, to sense the contributing structural modes. By using 6 - 8 sensors, this method could also target multiple radiation modes\(^1\). However, like the distributed piezoelectric film, this approach is highly dependent on the geometry. In order to sense specific structural modes the piezoelectric sensors need to be shaped appropriately. However, significant analysis of the structural and radiation modes is required to determine which modes are important and the required sensor shapes.

As previously described, many of the suggested methods of sensing for ASAC have had drawbacks due to the large number of sensors or sensor dependency on geometry. A new structural quantity, composite velocity, or \(V_{\text{comp}}^2\), which is now known as the weighted sum of squared gradients (WSSG) has recently been developed which, when sensed, provides similar control while avoiding these problems\(^1\).

WSSG has been shown to be a near uniform structural quantity on a vibrating, simply supported, rectangular plate. This uniformity allows for insensitivity to sensor location, reducing the analysis required in choosing an appropriate sensing location, a difficulty encountered when making other point measurements of vibration. The quantity itself, which is a sum of weighted spatial gradients, can be measured using an array of only four accelerometers, and has been shown to provide similar control to other standard ASAC objective functions such as volume velocity. The purpose of this paper is to derive WSSG for a rectangular plates with clamped boundary conditions and to demonstrate its feasibility as a simple, more robust, yet effective objective function for use in ASAC.
II. Development

The governing equation for a vibrating plate is given by

\[
D \left( \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \right) w(x, y) - m \omega^2 w(x, y) = F(x, y)
\]

(1)

where \(D\) is the bending stiffness, \(w\) is the displacement, \(\omega\) is the excitation frequency, and \(m\) is the mass per unit area\(^{13}\). The plate excitation is assumed to be time harmonic, the displacement and force expressions can be assumed to be multiplied by \(e^{i\omega t}\) although not stated explicitly. This is an assumption made throughout the remainder of this paper. Also, for the current formulation the plate will be assumed to be excited by a series of point forces, so that \(F(x, y) = \sum F_q \delta(x - x_q)\delta(y - y_q)\) with \(F_q\) indicating the \(q\)th applied force as a complex number, \(\delta(\cdot)\) the Dirac delta function, and \(x_q\) and \(y_q\) the location of the \(q\)th force. This expression models the shakers used to provide the experimental disturbance and control.

Although an exact analytical solution to Eq. (1) for clamped rectangular plates is not available, a method assuming the product of beam mode shapes as the eigenfunctions of the plate can be used to provide an approximate analytical solution\(^{14,15}\). This analytical solution is given by

\[
w(x, y) = \sum_{q} \sum_{m} \sum_{n} \frac{F_q \phi_m(x_q) \phi_n(y_q) \phi_m(x) \phi_n(y)}{D(I_1 I_2 + 2I_3 I_4 + I_5 I_6) - m \omega^2 I_2 I_6}
\]

(2)

where

\[
\phi_p(\xi) = C_1 \left( \frac{\lambda_p \xi}{L_\xi} \right) - \frac{C_1 (\lambda_p)}{S_1 (\lambda_p)} S_1 \left( \frac{\lambda_p \xi}{L_\xi} \right)
\]

(3)

\[
C_1(\xi) = \cosh(\xi) - \cos(\xi)
\]

\[
S_1(\xi) = \sinh(\xi) - \sin(\xi)
\]

and

\[
I_1 = \int_0^{L_x} \phi_m^4(x) \phi_m(x) dx \quad I_2 = \int_0^{L_y} \phi_n^2(y) dy
\]

\[
I_3 = \int_0^{L_x} \phi_m''(x) \phi_m(x) dx \quad I_4 = \int_0^{L_y} \phi_n''(y) \phi_n(y) dy
\]

\[
I_5 = \int_0^{L_y} \phi_n^4(y) \phi_n(y) dy \quad I_6 = \int_0^{L_x} \phi_m^2(x) dx
\]

The subscripts \(m\) and \(n\) indicate structural mode numbers, and the values for \(\lambda_p\), the eigenvalues of the system, are given by the characteristic equation
\[
\cosh(\lambda_p) \cos(\lambda_p) = 1
\]

\(\phi(\cdot)\) is the assumed shape function. The typical geometry and application of forces can be seen in Fig. 1. This method has been verified and shown to predict accurately the natural frequencies, when compared to the Rayleigh-Ritz method and experimentally, for a clamped rectangular plate\textsuperscript{15}.

A. Derivation of the Control Metric for Clamped Plates

Previously, composite velocity was defined\textsuperscript{1} as

\[
V_{\text{comp}}^2 = \alpha(w)^2 + \beta \left( \frac{\partial w}{\partial x} \right)^2 + \gamma \left( \frac{\partial w}{\partial y} \right)^2 + \delta \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2,
\]

where the choice of weights \(\alpha, \beta, \gamma,\) and \(\delta\) were important in deriving a uniform quantity for the simply-supported rectangular plate. However, it may be noted that due to the assumption of time harmonic vibration, the time derivative causes each of the terms to be scaled by a constant, \((j\omega)^2\), which has no effect on the algorithm’s ability to provide control using this metric. In this formulation the time derivative will be removed, making ‘composite velocity’ a misnomer. The new formulation is given as

\[
WSSG = \alpha(w)^2 + \beta \left( \frac{\partial w}{\partial x} \right)^2 + \gamma \left( \frac{\partial w}{\partial y} \right)^2 + \delta \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2,
\]

where WSSG stands for the weighted sum of spatial gradients.

While Eqs (5) and (6) are fundamentally the same with regards to control for both the time harmonic simply-supported and clamped plates the choice of weights changes with the boundary conditions and is dependent on the eigenvalues of the solution to Eq. (1). To determine the weights, the derivatives of Eq. (2) with respect to \(x\) and \(y\), as shown by Eqs.(7) – (9), are used:

\[
\frac{\partial w}{\partial x} = \sum_q \sum_m \sum_n \frac{F_q \phi_m(x_q) \phi_n(y_q) \phi_m'(x) \phi_n(y)}{D(I_1 I_2 + 2I_3 I_4 + I_5 I_6) - m\omega^2 I_2 I_6} 
\]

\[
\frac{\partial w}{\partial y} = \sum_q \sum_m \sum_n \frac{F_q \phi_m(x_q) \phi_n(y_q) \phi_m'(x) \phi_n(y)}{D(I_1 I_2 + 2I_3 I_4 + I_5 I_6) - m\omega^2 I_2 I_6} 
\]

and

\[
\frac{\partial^2 w}{\partial x \partial y} = \sum_q \sum_m \sum_n \frac{F_q \phi_m(x_q) \phi_n(y_q) \phi_m(x) \phi_n'(y)}{D(I_1 I_2 + 2I_3 I_4 + I_5 I_6) - m\omega^2 I_2 I_6} 
\]

where
\[
\phi_m'(x) = \frac{\lambda_m}{L_x} \left[ S_2 \left( \frac{\lambda_m x}{L_x} \right) - \frac{C_1(\lambda_m) C_1 \left( \frac{\lambda_m x}{L_x} \right)}{S_1(\lambda_m)} \right]
\]

(10)

\[
\phi_n'(y) = \frac{\lambda_n}{L_y} \left[ S_2 \left( \frac{\lambda_n y}{L_y} \right) - \frac{C_1(\lambda_n) C_1 \left( \frac{\lambda_n y}{L_y} \right)}{S_1(\lambda_n)} \right]
\]

(11)

and

\[
S_2(q) = \sinh(q) + \sin(q)
\]

Each of the WSSG terms for the first mode can be seen plotted in Fig. (2). Note how the first term has maximum values in the middle, the x and y terms have maximum values on the outside edges, and the cross derivative term has maximum values on the corners. By summing these terms, with the weights chosen to normalize the maximum values, a uniform field can be created.

Terms in Eq. (7) are scaled by a factor of $\frac{\lambda_m}{L_x}$ when the derivative is taken, as shown by Eq. (10). Likewise, Eqs. (8) and (9) are scaled by factors of $\frac{\lambda_n}{L_y}$ and $\frac{\lambda_m \lambda_n}{L_x L_y}$ as shown by Eqs. (10) and (11). These scaling factors are responsible for the difference in magnitudes of the terms. To create a uniform WSSG field, the inverse of these terms are chosen and squared to form the weights. These inverse expressions, as the clamped plate weights, are given in the first line of Table I. Note that each of the weights are scaled by $\alpha$. There is no specific value defined for $\alpha$, which can be used to scale all the terms by an equal amount. Normally the value of $\alpha$ is set to unity, however if the value were set so that $\omega = (j\omega)^2$, this formulation for WSSG becomes equivalent to Eq. (5), composite velocity.

An alternative way to define the weights is in terms of the wavenumber. The wavenumbers are inherent in the shape functions described by Eq. (3). For the clamped plate the wave number is $k = \frac{\lambda}{L}$, which can be substituted into the weights equations in the first line of the Table I to yield the general weights expressions shown on the second line of Table I. For the case of the simply-supported plate $k_m = \frac{m\pi}{L_x}$ which yields the correct weight expression $\beta_m = \left( \frac{L_x}{m\pi} \right)^2$. This can be demonstrated in a similar manner for $\gamma_n$ and $\delta_{mn}$, verifying that the general weight expressions are independent of boundary conditions and reduce down to the correct weights when the wavenumbers, corresponding to the chosen boundary condition, are used. This general expression is significant because it shows that each term is weighted by the corresponding wavenumber components, and allows for the the selection of weights on plates with non-ideal shapes if the wavenumber components in x and y can be measured.

At this point the derived weights are only valid for individual mode numbers $m$ and $n$, while the total displacement given in Eq. (2) is a sum over an infinite number of modes. While individual modes can be weighted analytically, this cannot be done experimentally, as the contributions to displacement from each of the modes cannot be separated in real time. Several different weighting schemes have been investigated for the non-mode specific weights with very similar results in simulated control. It can be concluded that the control metric is not overly sensitive to the general weighting scheme as long as they
are of the appropriate order of magnitude. The simplest method used was averaging the $\beta_m$, $\gamma_n$, and $\delta_{mn}$ values over the first 15 modes of the geometry examined.

B. Uniformity of WSSG Field

With the weights chosen as described above, the uniformity of the WSSG field on the clamped plate can be demonstrated. Using the equations shown in Table I, and the plate geometry and material properties defined in Table II, the WSSG field for the 1,2 mode is found by exciting the plate at the 1,2 mode natural frequency, and calculating the weights for and summing over only the 1,2 mode in Eqs. (2), (7), (8), and (9). This field is shown in Fig. (3), in decibels relative to the maximum value on the plate, to demonstrate uniformity. The inner black contour represents the boundary of the 3 dB down region, and contains approximately 35% of the plate. The outer black contour represents the boundary of the 6 dB down region, and contains approximately 50% of the plate. The standard deviation of the WSSG field is 13.3 dB.

A more practical case occurs when WSSG is summed over multiple modes (in Eqs. (2), (7), (8), and (9)), more closely modeling the displacement of an actual plate when it is excited. In this case the average weight values shown in Table I are used. The WSSG field summed over the first 15 modes using the average weights is shown in Fig. (4). In this case the 3 dB down region contains approximately 25% of the plate, and the 6 dB down region contains approximately 40% of the plate, so with more modes included the WSSG field becomes slightly less uniform. The standard deviation of the WSSG field in this case is 14.5 dB, a little higher than over a single mode.

When WSSG is summed over multiple modes the uniformity decreases, due to the contributions from other modes to the displacement of the plate. Also, the field on the clamped plate is less uniform than the WSSG field on the simply-supported plate due to the difference in boundary conditions. The clamped boundary prevents the plate from rotating at the edges, therefore driving the derivative terms to zero at the boundaries of the plate, which can be seen in Figs. (3) and (4). At higher modes and frequencies this transition area decreases. These results suggest that sensors ought to be placed away from the edges of the plate. Another important point to note is that while Figs. (3) and (4) show the WSSG field at the 1,2 natural frequency, there are no nodal lines in the field. In general nodal lines in the WSSG field do not appear regardless of the mode or frequency of excitation. This property significantly simplifies the choice of sensor location, reducing the importance of the exact location.

C. Comparison to Radiation Modes

One of the guiding factors in the original development of WSSG was the relationship between the radiation mode shapes and the shape of the spatial derivatives used. The radiation modes of a plate are derived from the radiation resistance matrix. The radiation resistance matrix discretizes the plate geometry and provides a simplified and computationally more efficient method for calculating the radiated sound power at low frequencies. It is derived by dividing the plate into elemental radiators.
and calculating the mutual radiation impedances between each elemental radiator and all others. This
matrix is given, for a rectangular plate\(^2\), by

\[
R = \frac{\omega^2 \rho_0 A_e^2}{4\pi c} \begin{bmatrix}
1 & \frac{\sin(kR_{12})}{kR_{12}} & \cdots & \frac{\sin(kR_{1N})}{kR_{1N}} \\
\frac{\sin(kR_{21})}{kR_{21}} & 1 & \cdots & \ddots \\
\vdots & \vdots & \ddots & \ddots \\
\frac{\sin(kR_{N1})}{kR_{N1}} & \cdots & \cdots & 1
\end{bmatrix}
\tag{12}
\]

where \(\rho_0\) is the density of the surrounding fluid, \(A_e\) is the elemental area, \(c\) is the speed of sound in the
fluid, \(R_{ij}\) is the distance from the \(i\)th to the \(i\)th element, and \(N\) is the total number of elements. Using
the radiation resistance matrix, the sound power is calculated\(^8\) as

\[
P = v_e^H R v_e
\tag{13}
\]

where \(v_e\) is a vector containing the velocity of each element, and \(H\) is the Hermitian transpose.

The radiation mode shapes are found as the eigenvectors of the radiation resistance matrix, and break
the matrix up into a set of mutually orthogonal vectors which reveal the underlying radiation
mechanisms of the structure. The radiation mode shapes, which are a function of geometry and
frequency, but not boundary conditions, generally only have a few of the structural modes contributing
significantly, thus by targeting the radiation modes the relevant radiating structural modes can be
targeted specifically. At low frequencies, it has been found that only a few of the radiation modes have
any significant contribution to sound radiation.

The first four radiation modes of the rectangular plate are shown in Fig. (6) and the four terms of WSSG
are shown in Fig. (7). Note the similarity between these four terms. The first radiation mode is similar in
shape to the displacement, \(w\), which is comparable to a monopole source; the second and third
radiation modes are similar to the derivatives with respect to \(x\) and \(y\), and can be compared to dipole
like radiation; the fourth radiation mode is similar to the cross derivative, and is similar to quadrupole
like radiation. These similarities, which were also found for the simply supported plate, suggest that
WSSG will be effective at targeting the first four radiation mode shapes for a clamped plate, and will
therefore be a good candidate for use in ASAC.

Because of the way in which the weights are calculated, a uniform WSSG field is created. The uniformity
of the field is considered an advantage as far as sensitivity to sensor placement is concerned, however it
can be viewed in light of weighting the radiation modes. By weighting each of the terms the same, each
of the radiation modes is given the same weight in the control algorithm, thus targeting each of the first
four radiation modes equally.
Using a scanning laser Doppler vibrometer (SLDV) the velocity of a plate of the same size and material properties as the simulations above, vibrating at the first natural frequency, was scanned. The displacement data were then used to calculate the various terms for WSSG and plotted. This can be seen in Fig. (5), which is very similar to Fig. (3), and therefore validates the theoretical development for WSSG as a uniform field over the plate.

**Numerical Simulations**

To demonstrate the effectiveness of WSSG, control was simulated on a clamped plate. The plate dimensions and material properties for the simulation are given in Table II. Equations (2), (6), (7), (8), and (9) were used as the analytical model for the plate. As stated previously point forces were used to excite and control the plate. The disturbance force was applied at the location $x_d = 0.083$ m, $y_d = 0.629$ m and the control force was applied at the location $X_c = 0.083$ m, $Y_c = 0.127$ m with WSSG measured at $x_s = 0.286$ m, $y_s = 0.432$ m. The control was simulated by finding the optimal control force for each frequency to minimize WSSG at the sensor location. The radiated sound power was then calculated by using elementary radiators as described in Eq. (13).

In order to demonstrate the effectiveness of WSSG it was compared to volume velocity, the most commonly used control metric for ASAC. For the analytical simulation volume velocity was found by discretizing the plate into 60 elements and estimating the velocity across each individual element as the velocity at the center. This number of elements was calculated as sufficient to give an accurate measure of volume velocity based on the methods described by Sors and Elliott\(^9\).

For the simulated control, shown in Fig. 7, the average attenuation over the frequency band from 0 - 200 Hz is 5.8 dB with WSSG as the control metric, and 7.3 dB with volume velocity as the control metric, so volume velocity, on average, outperforms WSSG by 1.5 dB. The majority of this difference is due to the significant amount of control which volume velocity achieves at both the first and third modes. At all other frequencies WSSG is similar to or better than volume velocity, and still provides significant control at these two modes. In particular note modes 2, 4, 5, 6, 7, 8, 11, 13, 14, and 15 which are controlled with WSSG but not with volume velocity. Volume velocity can't achieve control at these modes because they don't contribute significantly to the first radiation mode. However, WSSG can provide control at these modes because it also targets the second, third, and fourth radiation modes which these structural modes contribute to.

Also note the eighth and ninth modes. Neither metric can achieve control at this natural frequency. There are two possible reasons for this. This mode could be contributing to a radiation mode higher than mode 4, which was not targeted by either of the control metrics. Consider also that this mode is actually comprised of two closely spaced natural frequencies at 66 and 69 Hz which appear as a single degenerate mode, thus making it hard for a single control force to control this mode.

Control of the plate with the sensor placed at random locations was also simulated to determine control sensitivity to sensor location. Five locations were chosen at random on the plate and then control was simulated at those locations. A plot showing the control at these various locations is given in Fig. 8. Note the similarity in control regardless of the sensor location, in particular with regards to the modes noted previously. Regardless of location control of these modes can always be achieved, thus verifying that the uniform field created by choosing appropriate weights allows for the targeting of the first four radiation modes and their contributing structural modes, without sensitivity to the location of the sensor.
IV. Experimental Validation

A 6061-T6 rolled aluminum rectangular plate (19”x30”x0.122”) clamped on all 4 edges was used as the test structure. The clamped edge conditions were created by clamping the aluminum plate between two stiff frames with rows of bolts on all four sides.

Scans were performed on this plate using a Scanning Laser Doppler Vibrometer (SLDV) to ensure the plate demonstrated the no translation or rotation boundary condition used in the numerical model. The plate was excited with a Labworks ET-126 shaker attached to a signal generator and controlled with a Bruel and Kjaer type 4809 Vibration Exciter. These shakers were suspended from long cords and attached to the plate by gluing the individual stingers to the back side of the plate. WSSG was measured at a point using four accelerometers places one inch apart in a square configuration. A setup jig was used to ensure more accurate spacing of the accelerometers.

Control was achieved using a Filtered X LMS algorithm which accepted the four accelerometers as inputs and calculated WSSG using set weights programed into the code. A control signal was then sent through a filter and to the control Bruel and Kjaer shaker. The SLDV was used to measure the velocity at an array of points on the plate and then sound power was calculated using the method of radiation mode shapes.

The analytical model developed in this work was used to validate the experimental results. In the analytical model the test plate material properties, boundary conditions, excitation source placements and sensor placements were used. This allowed for a direct comparison between analytical models and experimental data. One thing which should be noted before a comparison between the two plots is made is that the two shakers used in the experimental results were fairly heavy (well over 10 pounds) and thus provided both a significant source of damping and mass loading. With the two shakers attached, a damping coefficient was measured using Logarithmic decrement to be approximately 0.03. Additionally, the mass loading of the plate caused the natural frequencies to shift down, often significantly. This frequency shift was proven by measuring the natural frequencies of the plate when the shakers were attached and when the plate was free. Thus when comparing the analytical results to the experimental results, it is necessary to compare the respective numerical equivalent mode number, and not compare the two plots frequency by frequency.

The first set of experimental results measured were for a situation where the excitation force was located at the x and y coordinates (as measured from the lower left corner) 0.083m and 0.629m, the control force located at 0.083m and 0.127m, and the sensor at 0.339m and 0.162 m. The computer simulation was made by calculating the sound power at each Hz with control on and off, while the experimental simulation was made by with measurements taken at each natural frequency, 5 Hz on either side of each natural frequency and at one point in between each natural frequency. This was judged enough to see the general trends of the experimental data.

Once the two plots were created (Fig. (8) and (9)), it was possible to compare the experimental results to the computer simulations over the first six natural frequencies and look for similar trends in the figures. The experimental results (Fig. (9)) matched the analytical results (Fig. (8)) with good degree of accuracy for the first four natural frequencies. There were significant differences at the fifth natural frequency and then nominal agreement again for the sixth natural frequency. The fifth mode was significant because the computer model showed significant attenuation at those frequencies, while the experimental results showed a minimal attenuation at the very peak itself, but then significant noise
amplification starts directly after the peak and continues for over 20 Hz. The cause of this discrepancy is currently being investigated.

Despite the differences between the two plots, the experimental results do show that controlling WSSG at a point can achieve significant noise attenuation and that the computer models do a nominally decent job of predicting actual results.

V. Conclusions

A weighted sum of spatial gradients control metric (WSSG) was developed for a clamped flat rectangular plate following the method used previously to develop composite velocity on a simply-supported flat rectangular plate. For a rectangular plate clamped on all four sides an exact analytical solution is not available, therefore an approximate solution assuming that the shape functions for the plate were the product of clamped-clamped beam shape functions was used. This solution was used to derive analytical expressions for the weights in WSSG, which caused the field to become relatively uniform. The development of WSSG with the choice of weights was intended to create a measurement quantity which was insensitive to sensor location, would target and control the first four radiation mode shapes, and could be measured in a much simpler manner experimentally than previous control metrics.

Simulated results verified the hypothesized effects. While only measuring the displacement at 4 locations on the plate comparable control to volume velocity (which required 16 - 25 accelerometers, or geometry dependent sensors) was achieved. In particular the control metric achieved improved control when compared to volume velocity at natural frequencies and modes higher than the fourth mode. This was due to the fact that the metric could control the second, third, and fourth radiation modes whereas volume velocity couldn't. The second, third, and fourth radiation modes have a greater contribution from structural modes with higher natural frequencies when compared to the first radiation mode, accounting for this improved control.

Also, the WSSG measurement sensor location was found to be relatively independent of position in simulations. This was shown by placing the sensor in random locations and comparing the control from those locations. Control was most consistent above the fourth mode natural frequency, where more of the targeted radiation mode shapes become relevant.

Finally, to demonstrate the effectiveness of WSSG experimental tests were performed. The experimental results were similar to the model verifying that the model was accurate and also confirming that the desired effects from the WSSG quantity do in fact occur.

From the observed effects it can be concluded that WSSG, which provides comparable control to volume velocity has several advantages in implementation. It requires fewer sensors, its physical measurement location is geometry independent, it works on multiple boundary conditions (demonstrated on simply-supported and clamped), and provides improved control over volume velocity at higher frequencies.
Figures:

**FIG. 1.** Coordinate system and force application on plate

(a) $w^2$ term  
(b) $\left(\frac{\partial w}{\partial x}\right)^2$  
(c) $\left(\frac{\partial w}{\partial y}\right)^2$  
(d) $\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2$

**FIG. 2.** WSSG terms for the first mode.
FIG. 3. WSSG field for the clamped plate 1,2 mode in dB, relative to the maximum value on the plate, at the 1,2 mode natural frequency. The inner black line represents the 3 dB down contour, and the outer the 6 dB down.
FIG. 4. WSSG field summed over the first 15 modes, at the 1,2 mode natural frequency, in dB relative to the maximum value on the plate. The inner black line represents the 3 dB down contour, and the outer the 6 dB down.
FIG. 5. SLDV measurement of WSSG field for clamped plate 1,1 mode in dB, relative to the maximum value on the plate, at the 1,1 natural frequency.
FIG. 6. The first four radiation mode shapes for a rectangular plate.
FIG. 7. Simulated control of the clamped plate using multiple control metrics

FIG. 8. Simulated control of the clamped plate at multiple sensor locations
FIG. 8. Predicted sound power levels for both clamped plate vibrating freely (Control off) and clamped plate when WSSG is being minimized at a point.

FIG. 9. Experimental sound power levels for both clamped plate vibrating freely (Control off) and clamped plate when WSSG is being minimized at a point.
TABLE I. Expressions for the weights in WSSG. The average weights shown were calculated using the geometry described in Table II.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$\partial w / \partial x$</th>
<th>$\partial w / \partial y$</th>
<th>$\partial^2 w / \partial x \partial y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clamped $\beta_m$</td>
<td>$\alpha \left( \frac{L_x}{\lambda_m} \right)^2$</td>
<td>$\gamma_n = \alpha \left( \frac{L_y}{\lambda_n} \right)^2$</td>
<td>$\delta_{mn} = \alpha \left( \frac{L_x L_y}{\lambda_m \lambda_n} \right)^2$</td>
</tr>
</tbody>
</table>

Weights

| General $\beta_m$ | $\alpha \left( \frac{1}{k_m} \right)^2$ | $\gamma_n = \alpha \left( \frac{1}{k_n} \right)^2$ | $\delta_{mn} = \alpha \left( \frac{1}{k_m k_n} \right)^2$ |

Weights

Avg. $\beta_{ave} = 0.00595$ $\gamma_{ave} = 0.00897$ $\delta_{ave} = 4.92 \times 10^{-5}$

Weights

TABLE II. Plate properties and dimensions for control simulations

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length in x ($L_x$)</td>
<td>0.483 m</td>
</tr>
<tr>
<td>Length in y ($L_y$)</td>
<td>0.762 m</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.0031 m</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>69.8 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.33</td>
</tr>
<tr>
<td>Density</td>
<td>2700 kg/m$^3$</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>2%</td>
</tr>
</tbody>
</table>
References


7 K. Naghshineh and G. H. Koopmann, “Active control of sound power using acoustic basis functions as surface velocity filters”, Journal of the Acoustical Society of America


